

Exercise 9.4 Escape rate of the tent map.

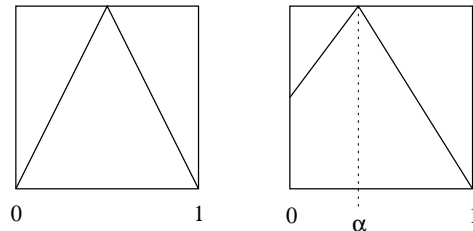
- (a) Calculate by numerical experimentation the log of the fraction of trajectories remaining trapped in the interval $[0, 1]$ for the tent map

$$f(x) = a(1 - 2|x - 0.5|)$$

for several values of a .³

- (b) Determine analytically the a dependence of the escape rate $\gamma(a)$.
 (c) Compare your results for (a) and (b).

Exercise 9.5 Invariant measure. We will compute the invariant measure for two different piecewise linear maps.



- (a) Verify the matrix \mathcal{L} representation (10.19).
 (b) The maximum value of the first map is 1. Compute an invariant measure for this map.
 (c) Compute the leading eigenvalue of \mathcal{L} for this map.
 (d) For this map there is an infinite number of invariant measures, but only one of them will be found when one carries out a numerical simulation. Determine that measure, and explain why your choice is the natural measure for this map.
 (e) In the second map the maximum occurs at $\alpha = (3 - \sqrt{5})/2$ and the slopes are $\pm(\sqrt{5} + 1)/2$. Find the natural measure for this map. Show that it is piecewise linear and that the ratio of its two values is $(\sqrt{5} + 1)/2$.

(medium difficulty)

Exercise 9.6 Escape rate for a flow conserving map. Adjust Λ_0, Λ_1 in (10.17) so that the gap between the intervals $\mathcal{M}_0, \mathcal{M}_1$ vanishes. Show that the escape rate equals zero in this situation.

Exercise 9.7 Eigenvalues of the Perron-Frobenius operator for the skew Ulam tent map. Show that for the skew Ulam tent map

³Mason: suggest specific values