16 Nearly ideal flow

16.1 Leonardo's law tells us that the average velocity on the top of the barrel is $v_0 = \frac{A}{A_0}v$, where v is the average velocity in the spout. Bernoulli's theorem now says

$$\frac{1}{2}v_0^2 + \frac{p_0}{\rho_0} + g_0 h = \frac{1}{2}v^2 + \frac{p_0}{\rho_0} \qquad \text{(16-A1)}$$

so that with $\kappa = A/A_0$

$$v = \sqrt{\frac{2g_0 h}{1-\kappa^2}} \qquad \text{(16-A2)}$$

The speed is slightly higher than the speed of free fall.

16.2 Take two streamlines that pass from the surface in the barrel through the two spouts. Then we have

$$\frac{p_0}{\rho_0} + g_0 h = \frac{1}{2}v_1^2 + \frac{p_0}{\rho_0} = \frac{1}{2}v_2^2 + \frac{p_0}{\rho_0} \qquad \text{(16-A3)}$$

or $v_1 = v_2$.

16.3 The solution to the differential equation (16-21) with initial condition $z = h$ at $t = 0$ is

$$z = \left(\sqrt{h} - \frac{1}{2}t\frac{A}{A_0}\sqrt{2g_0}\right)^2 = h\left(1 - \frac{t}{T}\right)^2 \qquad \text{(16-A4)}$$

where T is the total time (16-22) for emptying the barrel.

16.4 For an ideal gas $\rho = p/C$ where $C = RT_0/M_{\mathsf{mol}}$ and thus $w = C\log p$.
(a) Bernoulli's theorem becomes

$$\frac{1}{2}U^2 + C\log p_0 = C\log p\ , \qquad \text{(16-A5)}$$

which is solved for the pressure p in the tube,

$$p = p_0 \exp\left(\frac{M_{\mathsf{mol}}U^2}{2RT_0}\right) \qquad \text{(16-A6)}$$

(b) Taking $T_0 = 200$ K and $M_{\mathsf{mol}} = 29$ g/mol, we obtain $(p - p_0)/p_0 = 73\%$.

16.5 From Lenoardo's law we have

$$v_x = \frac{Q}{h(x) - b(x)} \qquad \text{(16-A7)}$$

where Q is a constant. From mass conservation, $\nabla_z v_z = -\nabla_x v_x$ it follows that $v_z = f(x) - z\nabla_x v_x$. At the bottom $z = b$ we have the boundary condition $v_z = v_x b'$ and at the surface $z = h$ we have $v_z = v_x h'$. Either of these determine $f(x)$.

