Self-Organization and Threshold of Stimulated Raman Scattering

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We derive, both theoretically and using an envelope code, threshold intensities for stimulated Raman scattering, which compare well with results from Vlasov simulations. To do so, we account for the nonlinear decrease of Landau damping and for the detuning induced by both the nonlinear wave number shift δk_p and the frequency shift $\delta\omega_p$ of the plasma wave. In particular, we show that the effect of δk_p may cancel out that of $\delta\omega_p$, but only in that plasma region where the laser intensity decreases along the direction of propagation of the scattered wave. Elsewhere, δk_p enhances the detuning effect of $\delta\omega_p$.

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In this Letter, we address the dynamical evolution of stimulated Raman scattering (SRS) of a laser in a plasma, in the so-called kinetic regime dominated by nonlinear wave-particle interactions. This requires understanding the self-organization of a complex nonlinear system, which is a major issue in modern physics (see, for example, Ref. [1]). Theoretical descriptions of self-organization are usually out of reach, despite notable attempts to use simple criteria (see Ref. [2] and references therein). This is, nevertheless, what we provide here for SRS, by assuming that it self-optimizes its growth, i.e., that the electron plasma wave (EPW) space profile self-organizes so as to cancel, as much as possible, the detuning induced by its nonlinear frequency shift $\delta\omega_p$. This result is akin to that of autoresonance, i.e., a space variation of the EPW amplitude such that nonlinear effects cancel out the detuning induced by inhomogeneity, as shown in Ref. [3]. However, autoresonance, as derived in Ref. [3] with prescribed nonlinear terms, cannot be straightforwardly extrapolated to a nearly homogeneous plasma since this would lead to infinitely small EPW amplitudes. This calls for a new theoretical framework, which allows for self-consistently derived nonlinear terms, and which we provide in this Letter. Although our theory is quite general, we restrict here to homogeneous plasmas because our main point is the modeling of nonlinear kinetic effects, which are more easily evidenced when the plasma density is uniform. More specifically, we derive and solve in this Letter envelope equations which allow for the nonlinear decrease of Landau damping and for the nonlinear frequency shift of the plasma wave, as derived in Refs. [4–7], but also for the self-optimization of SRS. As will be shown, accounting for each of these effects is necessary to make predictions in good agreement with those of Vlasov simulations as regards the growth of SRS, as can be seen in Figs. 2 and 3(b). This is particularly true to derive threshold intensities for SRS, i.e., conditions ensuring that Raman reflectivity remains very small, which is a major issue for inertial confinement fusion [8]. We, however, do not focus here on any specific application, since the scope of this Letter is to give a general description of the nonlinear kinetic effects and of their impact on SRS.

One of the main points of this Letter is the effect of $\delta\omega_p$ on SRS, which has actually been widely discussed in the past. Detuning due to the nonlinear EPW frequency shift was proposed in Ref. [9] as a saturation mechanism for SRS, while in Ref. [10], and for the same parameters as in Ref. [9], the growth of a sideband was shown to give rise to bursts in Raman reflectivity. The breakup of the EPW was further investigated in Ref. [11], and was attributed to modulational instability, induced by $\delta\omega_p$. Moreover, wave front bowing induced by $\delta\omega_p$ was evidenced in Ref. [11], and was experimentally shown to lead to an effective side-scatter in Ref. [12]. In this Letter, we focus on the detuning induced by $\delta\omega_p$ during the course of SRS growth, which has recently been questioned. Indeed, several recent publications, Refs. [13–15], pointed out the effect of the nonlinear plasma wave number shift δk_p, which could cancel out the detuning induced by $\delta\omega_p$. However, in Ref. [5], a closer look at the time evolutions of the nonlinear wave number shifts showed that this cancellation was not perfect and that a phase mismatch $\delta\varphi$ between the laser, scattered, and plasma waves did build up as SRS grew. However, no quantitative estimate of this effect was available at that time, and this is what we now address. As shown in Refs. [14,15], and is obvious from the envelope equation (5) below, cancellation of the effect of $\delta\omega_p$ by that of δk_p requires $\delta k_p = \delta\omega_p/v_{gs}$, where v_{gs} is the group velocity of the scattered wave which, so far, may be either positive or negative. Now, clearly, for δk_p to change from its initial value $\delta k_p \approx 0$ to $\delta\omega_p/v_{gs}$, while $|\delta\omega_p|$ keeps on increasing with time as SRS grows, $\partial_t \delta k_p$ needs to be of the same sign as $\delta\omega_p/v_{gs}$. As is well known, and will be proved further in this Letter, $\partial_t \delta k_p = -\partial_x \delta\omega_p$, where x is the direction of propagation of the waves. Hence, $\delta k_p = \delta\omega_p/v_{gs}$ requires $\partial_x \delta\omega_p(v_{gs}/\delta\omega_p) < 0$. Let us now assume, using the results of Refs. [5,16], that $\delta\omega_p$ only depends on the plasma wave amplitude E_p, and that $d\delta\omega_p/dE_P < 0$ and $\delta\omega_p < 0$. Then, the condition $\partial_x\delta\omega_p(v_{gs}/\delta\omega_p) < 0$ translates into $v_{gs}\partial_x E_p < 0$. Now, provided that the laser is focused inside of the plasma, as is usually the case in experiments, it is clear that the space profile of E_p should be similar to that of the laser intensity. We therefore conclude that only in that space domain where the laser intensity decreases along the direction of propagation of the scattered wave may δk_p cancel out the detuning effect of $\delta\omega_p$ while, elsewhere, it enhances it. As shown in Fig. 1, this effect is clearly observed in our Vlasov simulations described below.

In this Letter we study one-dimensional simulations of SRS, but we also want to somehow allow for the x dependence of the laser intensity which, as just shown, has important consequences on Raman scattering. To do so, we artificially multiply, in the equations we solve for SRS, the ponderomotive force $\vec{v} \times \vec{B}$ by a Lorentzian function $\mathcal{L}(x)$. Then, as shown in Fig. 1, and in agreement with our previous reasoning, wherever $v_{gs}\partial_x\mathcal{L}(x) < 0$, $|\Delta| \equiv |\delta\omega_p - v_{gs}\delta k_p| \ll |\delta\omega_p|$ and the phase mismatch $\delta\varphi$ remains small while, elsewhere, $|\Delta| > |\delta\omega_p|$ and $|\delta\varphi|$ increases nearly monotonically with time. Hence, $\delta\omega_p$ does induce a detuning that slows down the growth of SRS, but only in one half of the plasma. How to correctly account for this is one of the main points of this Letter.

Let us now detail our kinetic modeling of SRS, in the regime when the waves are nearly monochromatic, so that the total electric field may be written as $\vec{E}_{\text{tot}} = \mathcal{E}_p \sin(\varphi_p)\hat{x} + [\mathcal{E}_l \sin(\varphi_l) + \mathcal{E}_s \cos(\varphi_s)]\hat{y}$, where $\mathcal{E}_{p,l,s}$ are, respectively, the slowly varying amplitudes of the plasma, laser, and scattered waves, which are real and positive quantities. $\varphi_{p,l,s}$ are rapidly varying phases from which one defines the wave numbers $k_{p,l,s} \equiv \partial_x\varphi_{p,l,s}$ and frequencies $\omega_{p,l,s} \equiv -\partial_t\varphi_{p,l,s}$. Then, it is clear that $\partial_t k_{p,l,s} = -\partial_x\omega_{p,l,s}$. We define the phase mismatch between the three waves as $\delta\varphi \equiv \varphi_p + \varphi_s - \varphi_l$ and assume that, in the linear limit, $\delta\varphi = 0$. This amounts to assuming $\omega_l^{\text{lin}} = \omega_p^{\text{lin}} + \omega_s^{\text{lin}}$ and $k_l^{\text{lin}} = k_p^{\text{lin}} + k_s^{\text{lin}}$, where the superscript "lin" refers to the linear value of the considered quantity. Then, directly from Maxwell's laws, and using the results of Refs. [4–7], we derive the following envelope equations for the real wave amplitudes,

$$[\partial_t + v_{gp}\partial_x + \nu]\mathcal{E}_p = \mathcal{L}(x)(\Gamma_p/2)\mathcal{E}_l\mathcal{E}_s\cos(\delta\varphi), \quad (1)$$

$$[\partial_t + v_{gs}\partial_x - i\Delta_s]\mathcal{E}_s = (\Gamma_s/2)\mathcal{E}_l\mathcal{E}_p e^{-i\delta\varphi}, \quad (2)$$

$$[\partial_t + v_{gl}\partial_x - i\Delta_l]\mathcal{E}_l = -(\Gamma_l/2)\mathcal{E}_s\mathcal{E}_p e^{i\delta\varphi}, \quad (3)$$

where $\Gamma_{l,s} \equiv ek_p/(2m\omega_{s,l})$, $-e$ being the electron charge and m its mass; $v_{gl,s} \equiv k_{l,s}c^2/\omega_{l,s}$, with c being the speed of light in vacuum; and $\Delta_{l,s} \equiv [\omega_{l,s}^2 - (k_{l,s}c)^2 - \omega_{pe}^2]/2\omega_{l,s}$, where ω_{pe} is the electron plasma frequency. In Eq. (1), $\Gamma_p \equiv ek_p/(m\omega_l\omega_s\partial_\omega\chi_{\text{eff}}^r)$, where the derivation of $\partial_\omega\chi_{\text{eff}}^r$ and its nonlinear variations can be found in Refs. [4,6]; ν is the nonlinear Landau damping rate of the

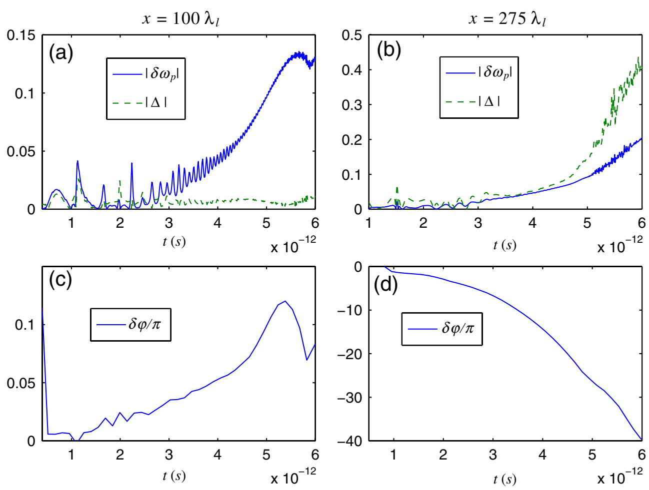

FIG. 1 (color online). Results from Vlasov simulations of Raman backscatter for an intensity profile given by the green dashed curve of Fig. 3(a), when $T_e = 5$ keV, $n/n_c = 0.1$, and $I_L = 4 \times 10^{15}$ W/cm^2, showing $|\delta\omega_p|$ [blue solid line in panel (a)] and $|\Delta| \equiv |\delta\omega_p - v_{gs}\delta k_p|$ [green dashed line in panel (a)] normalized to the plasma frequency when $x = 100\lambda_l$, i.e., when the laser intensity decreases along the direction of propagation of the backscattered wave. Panel (b) is the same as panel (a) but when $x = 275\lambda_l$, i.e., when the laser intensity increases along the direction of propagation of the backscattered wave; panels (c) and (d) are for $\delta\varphi/\pi$, when $x = 100\lambda_l$ and $x = 275\lambda_l$.

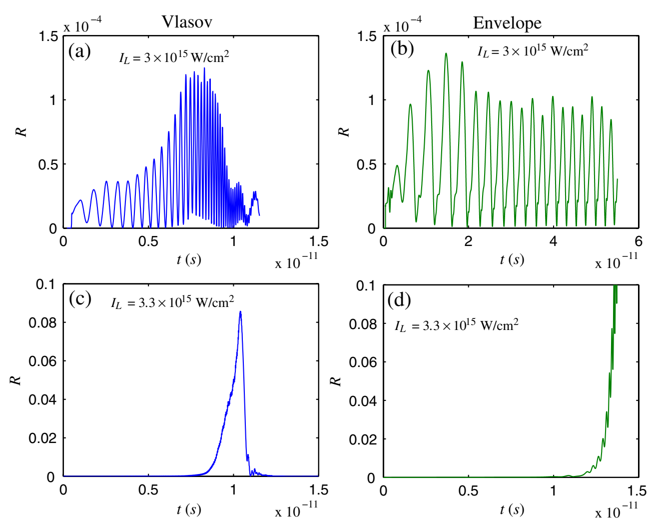

FIG. 2 (color online). Raman reflectivity as a function of time when $T_e = 5$ keV, $n/n_c = 0.1$, and $I_L = 3 \times 10^{15}$ W/cm^2 or $I_L = 3.3 \times 10^{15}$ W/cm^2. Panels (a) and (c) show calculations using Vlasov simulations, while panels (b) and (d) show those using our envelope code.

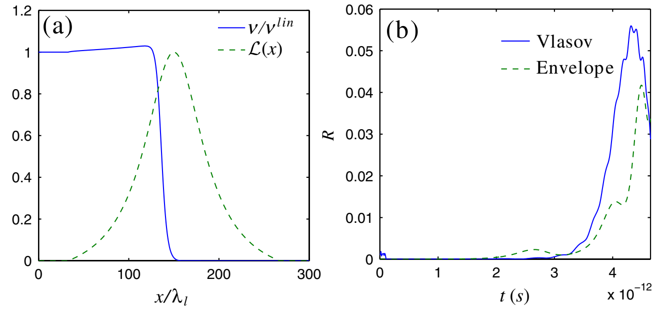

FIG. 3 (color online). Panel (a): Landau damping rate normalized to its linear value, as calculated by our envelope code when $T_e = 5$ keV, $n/n_c = 0.1$, $I_L = 3\times10^{15}$ W/cm^2, i.e., just below threshold, and at time $t = 20$ ps (blue solid line) and $\mathcal{L}(x)$ (green dashed line). Panel (b): Raman reflectivity when $T_e = 3$ keV, $n/n_c = 0.1$, $I_L = 1.25\times10^{15}$ W/cm^2 from Vlasov simulations (blue solid line) and from our envelope code (green dashed line).

EPW, whose explicit expression can be found in Ref. [6], and v_{gp} is the nonlinear group velocity of the plasma wave whose variations can also be found in Ref. [6]. Moreover, the factor $\mathcal{L}(x)$ in Eq. (1) results from our artificial multiplying of the ponderomotive force $\vec{v}\times\vec{B}$ by $\mathcal{L}(x)$, in order to account for the longitudinal space profile of the laser intensity. In order to use the criterion of self-optimization, which is a key ingredient in our theory, we shift to the following wave amplitudes: $E_{p,l,s}$, defined by $\mathcal{E}_p \equiv 2E_p$, $\mathcal{E}_l \equiv 2E_l e^{-i\delta\varphi_l}$, and $\mathcal{E}_s \equiv 2E_s e^{-i\delta\varphi_s - i\delta\varphi_p}$, where $\delta\varphi_{p,l,s} \equiv \varphi_{p,l,s} - (k^{\rm lin}_{p,l,s}x - \omega^{\rm lin}_{p,l,s}t)$ denote the nonlinear phase shift for each wave. Note that, so far, $E_{l,s}$ are complex quantities, while E_p is still real. In the limit when $\delta\omega_{l,s} \equiv (\omega_{l,s} - \omega^{\rm lin}_{l,s})$ and $\delta k_{l,s} \equiv (k_{l,s} - k^{\rm lin}_{l,s})$ are so small that $(\delta\omega_{l,s}/\omega_{l,s})^2$ and $(\delta k_{l,s}/k_{l,s})^2$ may be omitted in the envelope equations, Eqs. (1)–(3) readily yield

$$[\partial_t + v_{gp}\partial_x + \nu]E_p = \mathcal{L}\Gamma_p\, \mathrm{Re}(E_l E_s^*), \tag{4}$$

$$[\partial_t + v_{gs}\partial_x + i(\delta\omega_p - v_{gs}\delta k_p)]E_s = \Gamma_s E_l E_p^*, \tag{5}$$

$$[\partial_t + v_{gl}\partial_x]E_l = -\Gamma_l E_s E_p, \tag{6}$$

where $\mathrm{Re}(x)$ stands for the real part of x.

We want to use in Eq. (5) the adiabatic values of $\delta\omega_p$ derived in Ref. [5], which were shown to be quite accurate. Yet, despite their accuracy, these values are just not good enough because, since they only depend on the EPW amplitude and not on the dynamical evolution of SRS, they cannot allow for Raman self-optimization. As a result, plugging $\delta k_p(x,t) = \int_0^t -\partial_x\delta\omega_p(x,u)du$ into Eq. (5), with $\delta\omega_p$ calculated as in Ref. [5], leads to large threshold intensities and to a spurious rapid saturation of SRS due to dephasing which are not recovered in Vlasov simulations. An exact theoretical derivation of $\delta\omega_p$ being clearly out of reach, there is little hope for a direct exact resolution of Eqs. (4)–(6), and this is where our hypothesis of self-optimization enters. We keep the adiabatic values of $\delta\omega_p$ but plug into Eq. (5) $\delta k_p = \min(\int_0^t -\partial_x\delta\omega_p du, \delta\omega_p/v_{gs})$ when $\delta\omega_p/v_{gs} > 0$, and $\delta k_p = \max(\int_0^t -\partial_x\delta\omega_p du, \delta\omega_p/v_{gs})$ in the opposite case. Hence, we enforce the convergence of δk_p towards $\delta\omega_p/v_{gs}$, whenever the gradient of $\delta\omega_p$ allows it. This amounts to assuming that the EPW space profile self-organizes so as to cancel, as much as possible, the detuning induced by $\delta\omega_p$. Similarly, keeping E_p real while using for $\delta\omega_p$ our approximate adiabatic expressions leads to technical difficulties due to the fact that E_p may change sign. In order to avoid these difficulties, we need to allow the frequency shift of the EPW to assume slightly different values than our adiabatic ones, which amounts to allowing E_p to have a time-varying phase. To do so, we replace $\mathrm{Re}(x)$ with the identity operator in the right-hand side of Eq. (4). Hence, unless one is able to *exactly* calculate $\delta\omega_p$, deriving accurate envelope equations like Eqs. (4)–(6) (formally exact when the higher order space and time derivatives of the wave amplitudes are 0) is not enough. A practical resolution requires a slight change in these equations, and the use of the hypothesis of self-optimization.

In order to test the accuracy of our modeling, we now compare results derived from the numerical resolution of Eqs. (4)–(6) against those of Vlasov simulations of SRS. In all our simulations, the ions form a uniform neutralizing background, the electron temperature T_e and density n are uniform, and n is expressed in terms of the critical density, $n_c \equiv \varepsilon_0 m\omega_l^2/e^2$. We actually simulate the optical mixing of a laser with peak intensity I_L and of a counterpropagating seed with peak intensity $I_s = 10^{-5}I_L$. We therefore only make simulations of Raman backscatter. The laser wavelength is $\lambda_l = 0.351$ μm, while the seed wavelength is chosen so as to maximize the SRS growth rate in the linear regime. The total length of the simulation box is $L = 350\lambda_l \approx 123$ μm, and the Lorentzian function is $\mathcal{L}(x) = \{1 + [(x - x_0)/\Delta_x]^2\}^{-1} - \{1 + [(x_z - x_0)/\Delta_x]^2\}^{-1}$ when $x_z < x < 2x_0 - x_z$ and 0 elsewhere, and we chose in all our simulations $x_0 = 150\lambda_l$, $x_z = 33\lambda_l$, and $\Delta_x = 39\lambda_l$. Vlasov simulations are performed using the code ELVIS described in Ref. [17], while the envelope code solving Eqs. (4)–(6) will be described in a forthcoming publication, and uses classical numerical algorithms. We first focus on the threshold laser intensity $I_{\rm th}$, below which Raman amplification of the seed is nearly ineffective and SRS reflectivity R is very small. Figure 2 shows the sharp transition in R induced by a very small increase of the laser intensity near threshold, which is akin to that experimentally observed in Ref. [18]. Such a transition, for the threshold intensities we find, can only be explained by the nonlinear decrease of the Landau damping rate ν, as described in Ref. [6]. As shown in Table I, there is a good agreement between the values of $I_{\rm th}$ derived from our envelope code and from Vlasov simulations over a wide range of $k_p\lambda_D$, where λ_D is the Debye length. It is noteworthy that, just below threshold, in both Vlasov and

TABLE I. Values of $I_{\rm th}$ and I_3 expressed in 10^{15} W/cm^2 as a function of T_e, expressed in keV, and n/n_c.

$k\lambda_D$	T_e	n/n_c	Envelope	Vlasov	Theory
0.3	2	0.1	$0.225 < I_{\rm th} < 0.25$	$0.25 < I_{\rm th} < 0.3$	0.55
			$0.55 < I_3 < 0.6$	$0.55 < I_3 < 0.6$	
0.357	3	0.1	$0.7 < I_{\rm th} < 0.75$	$0.75 < I_{\rm th} < 1$	2
			$1.8 < I_3 < 2$	$1.8 < I_3 < 2$	
0.406	4	0.1	$1.8 < I_{\rm th} < 2$	$1.8 < I_{\rm th} < 2$	3.8
			$5 < I_3 < 5.5$	$3.5 < I_3 < 4.5$	
0.448	5	0.1	$3 < I_{\rm th} < 3.3$	$3 < I_{\rm th} < 3.3$	5.6
			$8 < I_3 < 9$	$6 < I_3 < 8$	
0.45	3	0.07	$2.5 < I_{\rm th} < 3.$	$2.7 < I_{\rm th} < 3$	3.8
			$7 < I_3 < 8$	$5 < I_3 < 6$	
0.513	4	0.07	$9 < I_3 < 11$	$9 < I_3 < 11$	
0.57	5	0.07	$12 < I_3 < 13$	$12 < I_3 < 15$	

envelope simulations, SRS reflectivity oscillates with time, thus evidencing the detuning effect of $\delta\omega_p$. Moreover, R may actually be significantly larger than the value $R^{\rm lin} = 1.5 \times 10^{-5}$ linear theory would predict, which shows that ν has been nonlinearly reduced, at least in a substantial fraction of the simulation box. Actually, from our envelope simulations, and as shown in Fig. 3(a), we find that just below threshold for Raman backscatter, ν assumes its linear value in that space domain before the maximum of $\mathcal{L}(x)$, i.e., when $x < x_0$, and is 0 elsewhere. The physical interpretation of this result is quite clear. In a situation such as that of Fig. 3(a), SRS is ineffective when $x < x_0$ because the EPW is too Landau damped, but also when $x > x_0$ because of the detuning effect of $\delta\omega_p$ enhanced by that of δk_p. A further increase of the laser intensity would lead to a nonlinear reduction of ν in the space domain $x < x_0$, where SRS is not hampered by the effect of $\delta\omega_p$, and therefore to a large increase of the reflectivity. Using this result, we now derive a theoretical estimate for $I_{\rm th}$. From the results of Ref. [6], we know that $\nu \approx 0$ whenever $\int \omega_B dt > 6$, where $\omega_B \equiv \sqrt{eE_p k_p/m}$, and where the integral is calculated in the plasma wave frame. We therefore estimate that, at threshold, $\int_0^{x_0} (\omega_B/v_\phi) dx = 6$, where v_ϕ is EPW phase velocity. Since, at threshold, ν assumes its linear value when $x < x_0$, we use linear theory in the strong damping limit, and in a steady state, to evaluate E_p there. Hence, when $x < x_0$, we estimate $E_p \approx \mathcal{L}\Gamma_p E_l E_s/\nu^{\rm lin}$, with $E_s(x) = E_s(L)e^{[\int_L^x \gamma(\xi)d\xi]}$, where $\gamma(x) \equiv \mathcal{L}(x)\Gamma_p\Gamma_s|E_l|^2/(\nu^{\rm lin} v_{gs})$. Note that the latter expression for $E_s(x)$ underestimates its value at threshold because it does not account for the nonlinear reduction of ν when $x > x_0$. As a result, our theoretical values for $I_{\rm th}$ overestimate those derived from Vlasov simulations, but only by a factor close to 2, which is remarkable for such a simple calculation.

Close to threshold, it may take tens of picoseconds for SRS reflectivity to reach its first maximum so that Raman reflectivity could effectively be suppressed by such techniques as smoothing by spectral dispersion [8]. We therefore find it very important to check that we correctly model the time evolution of SRS reflectivity above threshold. To this end, we define I_3 as the laser intensity such that, if $I_L > I_3$, it takes less than 3 ps for SRS reflectivity to reach either 10% or the value of the first maximum found in Vlasov simulations. Table I shows that, for the seven cases we investigated, and which span values of $k_p\lambda_D$ ranging from 0.3 to 0.57, there is a good agreement between the values of I_3 found from our model and from Vlasov simulations. How accurately our model equations predict the growth of SRS may, moreover, be appreciated in Figs. 2 and 3(b).

In conclusion, we derived coupled envelope equations for SRS, allowing for nonlinear kinetic effects, and solved them using, as a basic ingredient, the hypothesis of self-optimization of simulated Raman scattering. This let us derive threshold intensities and SRS growth times in very good agreement with those found using Vlasov simulations, over a large range of $k_p\lambda_D$. We, moreover, came to the unexpected result that the nonlinear phase mismatch between the waves depended on the longitudinal profile of the laser intensity, and was small wherever $v_{gs}\partial_x I_L < 0$. This Letter thus provides a precise description of the impact of nonlinear kinetic effects on the growth of SRS.

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