

The second term vanishes because of the lack of correlation.

(b) The average of the second term is

$$\left\langle \frac{1}{2} \sum_n m_n (\mathbf{v}'_n)^2 \right\rangle = \frac{3}{2} MU^2 \quad (17-A26)$$

18 Viscosity

18.1 (a) In a unit volume, the ideal gas law tells us that the number of moles is p/RT , and since there is N_A molecules in a mole, the result follows. For $T = 300$ K and $p = 1$ bar, we get $n \approx 2.4 \times 10^{25} \text{ m}^{-3}$.

(b) A single molecule of diameter d will collide with a molecule of the same size if it gets inside a circle of radius d centered at the target molecule. In a cylinder with radius d and length equal to the mean-free-path λ , there must be exactly one molecule, or $n\lambda\pi d^2 = 1$, where n is the density of molecules. For air this becomes $\lambda \approx 147$ nm.

(c) For air the speed of sound is $c \approx 330$ m/s and the viscosity estimate becomes $\eta \approx \rho\lambda^2/\tau \approx \rho\lambda c \approx 5 \times 10^{-5}$ Pas, which is a factor 2 too large.

18.2 In an isentropic gas we have $p \sim \rho^\gamma$ and $p \sim \rho T$, so that $\rho \sim T^{1/(\gamma-1)}$ and $\nu \sim T^{1/2-1/(\gamma-1)}$. For monatomic gases $\gamma = 5/3$ and $\nu \sim T^{-1}$, for diatomic $\gamma = 7/5$ and $\nu \sim T^{-2}$, and for multiatomic $\gamma = 4/3$ and $\nu \sim T^{-5/2}$.

18.3 One finds $d \approx 3$ μm and $t_0 \approx 11$ s. The layer seems a bit thin compared to, for example curling (example 18.2.1). The tire pattern probably influences the “ice grip” considerably.

18.4 (a) The total flux is

$$Q(t) = \int_{-\infty}^{\infty} v_x(y, t) dy . \quad (18-A1)$$

From (18-5) we get by integrating over y

$$\frac{dQ}{dt} = \nu \int_{-\infty}^{\infty} \frac{\partial^2 v_x}{\partial y^2} dy = 0 , \quad (18-A2)$$

because $\partial v_x/\partial y$ vanishes at infinity.

(b) The total momentum per unit of length is

$$\mathcal{P} = \int_{-\infty}^{\infty} \rho_0 v_x(y, t) dy = \rho_0 Q \quad (18-A3)$$

and is constant because Q is.

(c) The kinetic energy per unit of length is

$$\mathcal{T} = \frac{1}{2} \rho_0 \int_{-\infty}^{\infty} v_x(y, t)^2 dy \quad (18-A4)$$

In the Gaussian case this becomes

$$\mathcal{T} = \frac{1}{2} \rho_0 \frac{a^2}{a^2 + 2\nu t} \int_{-\infty}^{\infty} \exp\left(-\frac{y^2}{a^2 + 2\nu t}\right) dy \quad (18-A5)$$

$$= \frac{1}{2} \rho_0 \sqrt{\pi} \frac{a^2}{\sqrt{a^2 + 2\nu t}} \quad (18-A6)$$