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Yang-Mills Theory on the Mass Shell*

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Gauge-invariant mass-shell amplitudes for quantum electrodynamics (QED) and Yang-Mills theory are defined by dimensional regularization. Gauge invariance of the mass-shell renormalization constants is maintained through interplay of ultraviolet and infrared divergences. Quark renormalizations obey the same simple Ward identity as do the electron renormalizations in QED, while the gluon contributions to gluon renormalizations are identically zero. The simplest amplitude finite in QED, the magnetic moment, is gauge-invariant but divergent in Yang-Mills theory for both external gluon and external photon.

It is traditional to treat ultraviolet (uv) and infrared (ir) divergences of quantum electrodynamics (QED) as separate problems. uv divergences are associated with the internal topology of Feynman diagrams, and they can be removed by a (possibly intermediate) renormalization. ir divergences are controlled by the external momenta, and they are traditionally regularized by the introduction of a photon mass, i.e., an abandonment of gauge invariance in the intermediate stages of calculation of physical cross sections. Even though ir and uv divergences thus appear quite unrelated, there are hints to the contrary. A systematic analysis of Feynman parametric integrals reveals a close connection between the two types of divergences[1]; furthermore, these divergences can be transmuted one into another by a change of gauge (for example, from Landau to Yennie gauge).

The breaking of gauge invariance through the introduction of a photon mass is acceptable for QED, but unacceptable[2] for Yang-Mills[3,4] theories. However, the dimensional regularization[5,6] makes it possible to regularize both uv and ir divergences of QED while keeping the photon strictly massless. I shall here first reconsider QED in this approach, concentrating on the regularization of ir divergences, and then use the same regularization procedure to give an unambiguous definition of Yang-Mills amplitudes on the mass shell. To stress the analogy with QED, I shall refer to the class of Yang-Mills theories considered here (symmetric, with all quarks of equal mass $m \neq 0$ and strictly massless gluons) as quantum chromodynamics (QCD). The details of the calculations will be published elsewhere.[7]

QED.—As their first example of dimensional regularization Bollini and Giambiagi[5] have computed one-loop contributions to electron vertex and wave-function mass-shell renormalizations $Z_1=(1+L)^1$ and $Z_2=(1-B)^{-1}$ in $4-\epsilon$ dimensions,

$$L=-B=-\frac{\alpha}{4\pi}\Gamma\left(\frac{\epsilon}{2}\right)\frac{-3+\epsilon}{1-\epsilon}, \tag{1}$$

where $\alpha \equiv [e_0^{\,2}/(4\pi)^{1-\epsilon/2}]m^{-\epsilon}$, and throughout this paper I set $m=1$. It can be verified by calculation in the generalized Landau gauge $[g^{\mu\nu}k^2-(1-a)k^\mu k^\nu]$ that this is gauge-independent.[7,8] That the gauge invariance persists to all orders can be verified either by a dimensional-regularization re-evaluation of Z_2 gauge dependence given by Johnson and Zumino,[9] or by standard combinatorics with Feynman current conservation identities[10]

$$\frac{1}{\not p+\not q-m}q\frac{1}{\not p-m}=\frac{1}{\not p-m}-\frac{1}{\not p+\not q-m}\,. \tag{2}$$

The second approach makes it clear that *all mass-shell amplitudes in QED are gauge invariant.* In the above calculations ir divergences are associated with integrals of form

$$\int_0^1 dx/x^{1+\epsilon} \tag{3a}$$

(Feynman parametric space), and

$$\int d^{4-\epsilon}k/k^4 \tag{3b}$$

(gauge independence in momentum space). (3a) is defined by the *analytic continuation* from $\epsilon<0$ (i.e., from dimensions higher than four). Intuitively, above four dimensions an x^{-2} potential has a finite range, so I am defining QED as a limit of an ir-finite theory. (3b) is the standard dimensional-regularization integral evaluated for $\lambda=0$, $\epsilon<0$:

$$\int\frac{dk^{4-\epsilon}}{(k^2-\lambda^2)^2}=i(-\pi)^{2-\epsilon/2}\Gamma\left(\frac{\epsilon}{2}\right)(-\lambda^2)^{-\epsilon/2}=0\,.$$

It will soon be shown that this amounts to a cancellation between ir and uv divergences. To summarize, in the dimensional-regularization scheme, ir divergences are regularized by

$$\int_0^1\frac{dx}{x^{1+\epsilon}}=-\frac{1}{\epsilon} \tag{4a}$$

(Feynman parametric space) and

$$\int\frac{dk^{4-\epsilon}}{k^4}=0 \tag{4b}$$

(momentum space). Any other definition of the above integrals introduces gauge dependence into mass-shell amplitudes.

QCD.—A new feature of QCD is the factorization of a Feynman integral into a group-theoretic weight W and a momentum integral M of QED type.[11] The weights are related by Lie-algebra

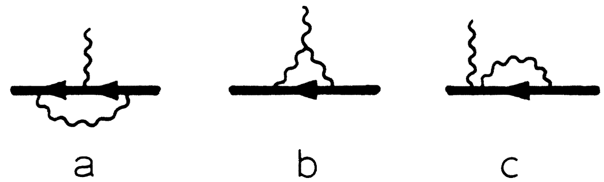

FIG. 1. (a) Vertex diagram with external gluon (photon) attached to the quark (electron) line; (b) vertex diagram with a three-gluon coupling; (c) self-energy diagram.

commutators; in Fig. 1 for example, $W_c-W_a=W_b$, and the momentum integrals for Figs. 1(a) and 1(c) are already given by (1). For Fig. 1(b), the explicit calculation in the generalized Landau gauge yields

$$L_b=L_a=-B_c=-\tfrac{1}{4}\Gamma(\tfrac{1}{2}\epsilon)\frac{-3+\epsilon}{1-\epsilon}\,,$$

so that the quark renormalizations satisfy the same simple Ward identity as the electron renormalizations in QED.

$$L+B=0\,, \tag{5}$$

with $B=(\alpha/\pi)W_cB_c$, $L=(\alpha/\pi)(W_aL_a+W_bL_b)$. The ir divergences of L_b have been regularized by (4a). It is important to note that L_b is ir-finite in Feynman gauge, while L_a and B_c are not, and that a λ cutoff could not yield (5).

Were (5) evaluated off the mass shell, it would have a nonvanishing, gauge-dependent contribution from Taylor ghost[12] diagrams on the right-hand side. On the mass shell, such diagrams vanish as a result of quark current conservation identities (2) and the analogous identities for momentum insertions into three- and four-gluon vertices given by 't Hooft.[13] These suffice to prove (5) and, in general, the gauge invariance of QCD mass-shell amplitudes to all orders. For Z_2, I have explicitly verified the gauge invariance (in covariant gauges) to the two-loop level. Again, all the gauge dependence is eliminated by the vanishing of integral (4b).

The radically new aspects of QCD are the gluonic corrections to the gluon propagator. In the Schwinger parametric representation,[14] the one-loop contribution is of the form

$$\Pi_{\mathrm{gluon}}{}^{\mu\nu}(q^2)=(q^\mu q^\nu-g^{\mu\nu}q^2)i^{\epsilon/2}\int_0^\infty\frac{dz_1dz_2}{z_{12}{}^{2-\epsilon/2}}I\exp\left(iq^2\frac{z_1z_2}{z_{12}}\right)\,, \tag{6}$$

where $z_{12}=z_1+z_2$, and I is a known polynomial[7] in z_1/z_{12}, z_2/z_{12}, a, and ϵ. I write $\Pi_{\mu\nu}$ in this form to illustrate the origin of ir-uv cancellation of type (4). Here the uv divergences arise from the $z_{12}\to 0$

region of integration, while the potential ir divergences from $z_1 \to \infty$ and $z_2 \to \infty$ are damped by $q^2 \neq 0$. Introduction of an overall scale $z_1 \to zz_1$ and $z_2 \to zz_2$, with $z_1 + z_2 = 1$,

$$\Pi_{\text{gluon}}(q^2) = i^{\epsilon/2}\int_0^1 dz_1 dz_2\,\delta(1 - z_1 - z_2) I \int_0^\infty \frac{dz}{z^{1-\epsilon/2}} \exp[-iz(-q^2 z_1 z_2)]. \qquad (7)$$

If $q^2 \neq 0$, the z integral has only an uv singularity and can be defined by analytic continuation from $\epsilon > 0$. The result is the Feynman parametric[14] representation of (6):

$$\Pi_{\text{gluon}}(q^2) = \frac{1}{(-q^2)^{\epsilon/2}}\Gamma\left(\frac{\epsilon}{2}\right)\int_0^1 dz_1 dz_2\,\delta(1 - z_{12})\frac{I}{(z_1 z_2)^{\epsilon/2}}.$$

However, if $q^2 = 0$, (7) is both uv and ir singular, but it has a unique definition

$$\int_0^\infty \frac{dz}{z^{1-\epsilon}} \equiv \int_0^1 \frac{dz}{z^{1-\epsilon}} + \int_1^\infty \frac{dz}{z^{1-\epsilon}} = 0, \qquad (8)$$

where the first (uv-divergent) integral is continued from $\epsilon < 0$. Because of the lack of an intrinsic mass scale, pure–Yang-Mills-field ir and uv divergences exactly cancel each other. This insures that even though the off-mass-shell $\Pi(q^2)$ is gauge-dependent, the mass-shell wave-function renormalization constant $C_{\text{gluon}} \equiv -\Pi_{\text{gluon}}(0)$ is gauge-independent, i.e.,

$$C_{\text{gluon}} = 0, \qquad (9)$$

and similarly, gluon contributions to the three-gluon vertex renormalization are vanishing. However, vanishing of C_{gluon} does not mean that the gluon propagator does not get renormalized because $Z_3 = (1 - C)^1$ picks up nonvanishing contributions from quark loops, just as in QED.

If the symmetry is broken and gauge mesons acquire a mass, the ir divergences will disappear, and the gauge dependence of the uv divergences, which are unaffected by the internal masses, will not be canceled. Hence, the above arguments do not apply to spontaneously broken gauge theories nor to the off-mass-shell renormalized QCD.

Magnetic moment.—As we have seen, it is possible to define QCD amplitudes and renormalizations on the mass shell. The interesting question is whether such mass-shell QCD allows any finite, physically measurable quantities. In QED the simplest such quantity is the anomalous magnetic moment. In QCD the explicit computation of Fig. 1(b) to the color magnetic moment of a quark yields an ir-singular answer.[15]

$$\tfrac{1}{2}(g-2)_b = -\frac{\alpha}{4\Pi} W_b \Gamma\left(\frac{\epsilon}{2}\right) \times \frac{-2-\epsilon}{1-\epsilon} \text{ (gague invariant)}.$$

For gluon corrections to the electromagnetic magnet moment of the quark on the one-loop level [Fig. 1(a) with an external photon and an internal gluon], the momentum integral is just the Schwinger correction.[16] However, the two-loop level yields an ir-singular (and gauge-invariant) magnetic moment as has been shown by Korthals Altes and de Rafael[17] and by the present author.[7] This persists to all orders.[18]

In summary, dimensional regularization provides an unambiguous definition of QED and QCD mass-shell amplitudes. The physical interpretation of such a QCD theory is discussed elsewhere.[18]

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Evidence for Parity Nonconservation in the Decays of the Narrow States near 1.87 GeV/c^2*

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We have studied the Dalitz plot for the recently observed charged state decaying into $K^{\mp}\pi^{\pm}\pi^{\pm}$ at 1876 MeV/c^2 and we find that the final state is incompatible with a natural spin and parity assignment. This information, coupled with the earlier observation of the $K^{\pm}\pi^{\mp}$ decay mode (a final state of natural spin and parity) of the neutral state at 1865 MeV/c^2, suggest parity nonconservation in the decays of these objects if they are members of the same isomultiplet as their proximity in mass suggests.

We have recently reported our observation in e^+e^- annihilation of a narrow, charged state of mass 1876 MeV/c^2 decaying into the exotic decay mode $K^{\mp}\pi^{\pm}\pi^{\pm}$.[1] The proximity in mass of this state to the neutral state decaying into $K\pi$ and $K3\pi$ at 1865 MeV/c^2 suggests that they are members of the same isomultiplet. As such they are expected to have the same parity. Since the $K\pi$ final state is one of natural spin and parity, a demonstration that the $K\pi\pi$ final state of the charged member of the isomultiplet is inconsistent with natural spin and parity implies parity nonconservation in the decay. In this Letter we present evidence, based on a study of the $K^{\mp}\pi^{\pm}\pi^{\pm}$ Dalitz plot, for such parity nonconservation, suggesting that the decay proceeds via the weak interaction as expected for the (D^+, D^0) isodoublet of charm.[2]

The present analysis is based on $K\pi\pi$ events observed among a sample of ~ 44 000 hadronic events taken from 3.9- to 4.25-GeV center-of-mass energy. These data were taken with the Stanford Linear Accelerator Center–Lawrence Berkeley Laboratory magnetic detector at SPEAR.

The $K\pi\pi$ combinations are selected with the aid of the time-of-flight system described in Goldhaber *et al.*[3] In the present analysis we have used a modified form of the time-of-flight (TOF) weighting technique described earlier.[1,3] A given track in a multiprong hadronic event is assigned a definite particle identity on the basis of the agreement between its observed TOF over a 1.5–2.0-m flight path and that predicted for either a π or a K with a momentum as measured. Specifically we compute a χ^2 value for both the π and K hypotheses (χ_π^2 and χ_K^2) based on the observed and expected TOF and the 0.4-ns rms resolution of the TOF system. Tracks satisfying the requirements $\chi_k^2 < \chi_\pi^2$, $\chi_K^2 < 3$, are called kaons. Protons and anitprotons are separated from kaons in a similar fashion. The remaining tracks are called pions.[4] The above technique allows the direct study of scatter plots and in particular the Dalitz plot for the $K\pi\pi$ system.

In order to obtain a relatively clean sample of $K\pi\pi(1876)$ events we make use of the result that for the $E_{\text{c.m.}}$ region $3.9 < E_{\text{c.m.}} < 4.25$ GeV, the recoil mass (M_{rec}) spectrum shows a sharp spike near 2 GeV.[1] We thus used a data sample with the $E_{\text{c.m.}}$ region chosen as above coupled with a cut $1.96 < M_{\text{rec}} < 2.04$ GeV/c^2. Figures 1(a) and 1(b) show the resulting exotic and nonexotic $K\pi\pi$ invariant-mass distributions. A fit to the spectrum of Fig. 1(b) was appropriately scaled to serve as a background for Fig. 1(a). Figure 1(a) shows a fit to a Gaussian peak over this back-