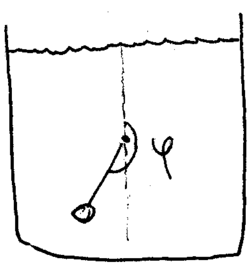
Homework set #1

2.1.5

a) A mechanical system approximately governed by $\dot{x} = \sin x$ is the overdamped pendulum, e.g. a simple pendulum immersed in a very viscous fluid.



In this case the equation of motion reads

$$m \ddot{\varphi} = m g \sin \varphi - \gamma \dot{\varphi}$$

where γ is proportional to the viscosity of the fluid. If $m \ll \gamma$, then $m \ddot{\varphi}$ can be neglected and the equation becomes

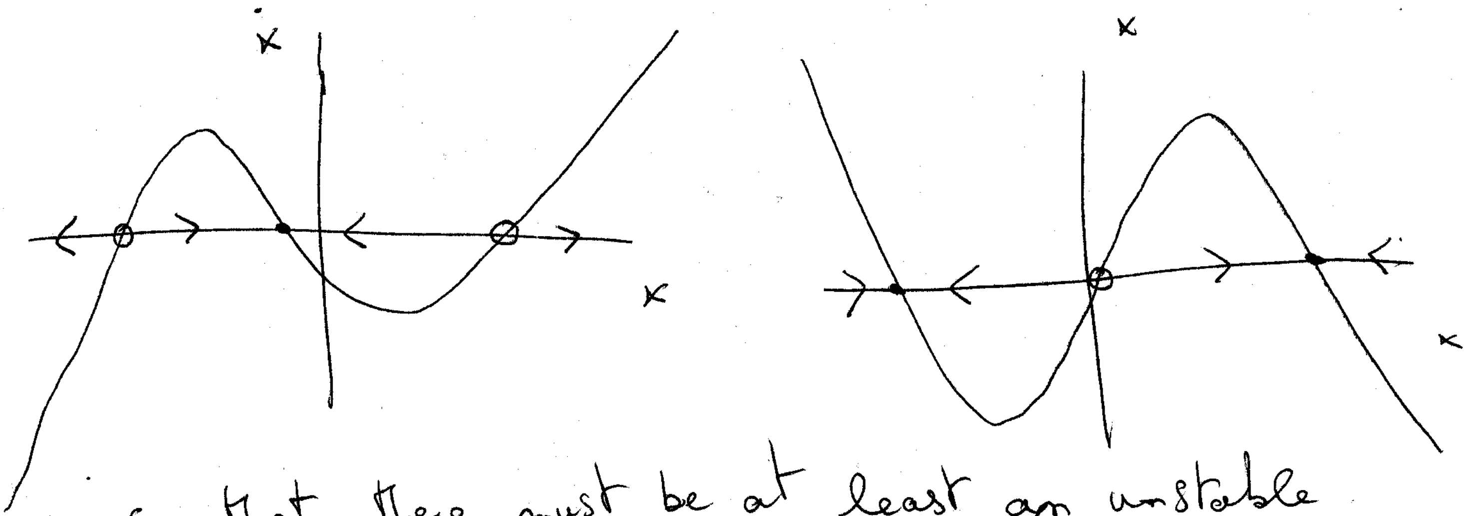
$$\sin \varphi \cong \frac{\gamma}{mg} \dot{\varphi}$$

b) It can be seen from the picture that the mass moves away from $\varphi_0 = 0$ (top) and approaches the stable equilibrium position at $\varphi = \pi$.

2.2.10

a) $f(x) = 0$; b) $f(x) = \sin(\pi x)$ or $\cos\left(\frac{\pi}{2}x\right)$

c) the only two possible configurations of the three roots are



so that there must be at least an unstable fixed point.

d) $f(x) = \text{const}$, $f(x) = e^{g(x)}$, $f(x) = 1 + x^{2n}$

e) Any polynomial $P(x)$ of degree 100 or higher which have 100 real and distinct roots

2.3.3

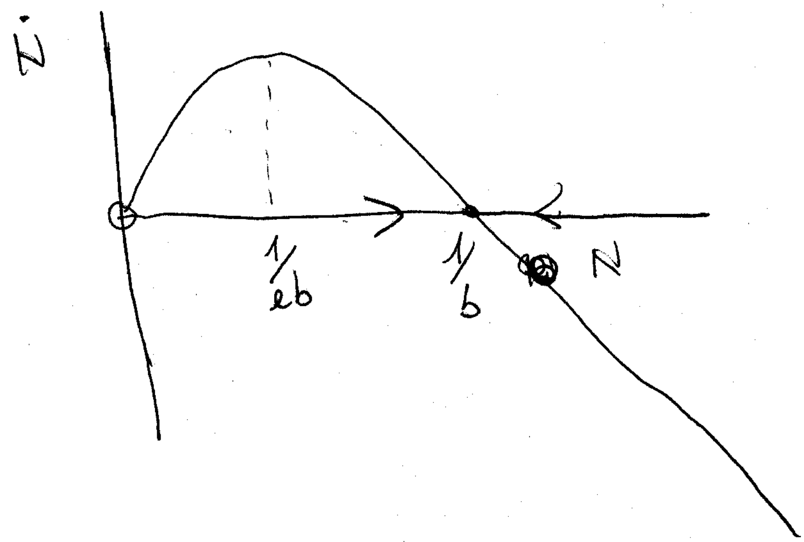
a) "a" is propotional to the rate of reproduction of the tumor cells, it determines how fast the tumor grows.

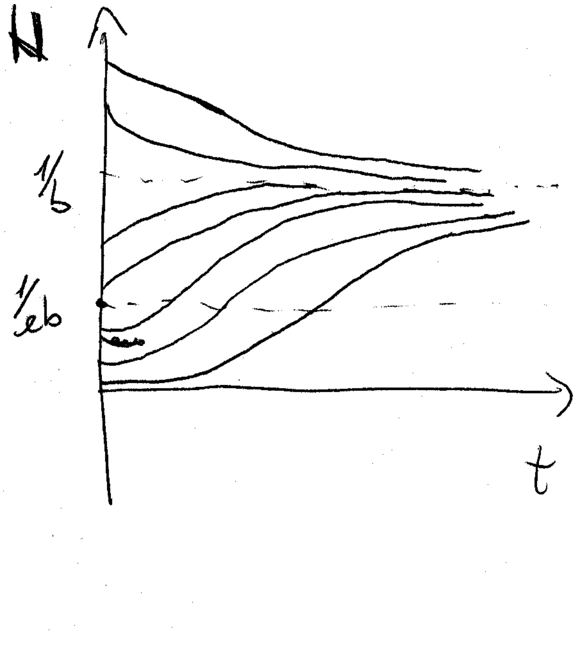
$f(N) = -aN\ln(bN) = 0$ if $N = \frac{1}{b}$.

As shown below, that is a stable fixed point, that corresponds to the number of tumor cells at the steady state, so that b determines in practice how big the tumor is.

b) $f(N) = -aN\ln(bN)$; $f'(N) = -a\left[\ln(bN)+1\right] = 0$

if $N_* = \frac{1}{eb}$, that is N_* is a maximum of the rate of growth:





HW #2

2.4.2 $\dot{x} = x(1-x)(2-x)$

$f'(x) = (1-x)(2-x) - x(2-x) - x(1-x)$

fixed points at $x = 0, 1, 2$

$f'(0) = 2$ unstable

$f'(1) = -1$ stable

$f(2) = 2$ unstable

2.4.9 a) $\dot{x} = -x^3$

$$\Rightarrow \int_{x_0}^{x} dx \cdot x^{-3} = -\int_0^t dt$$

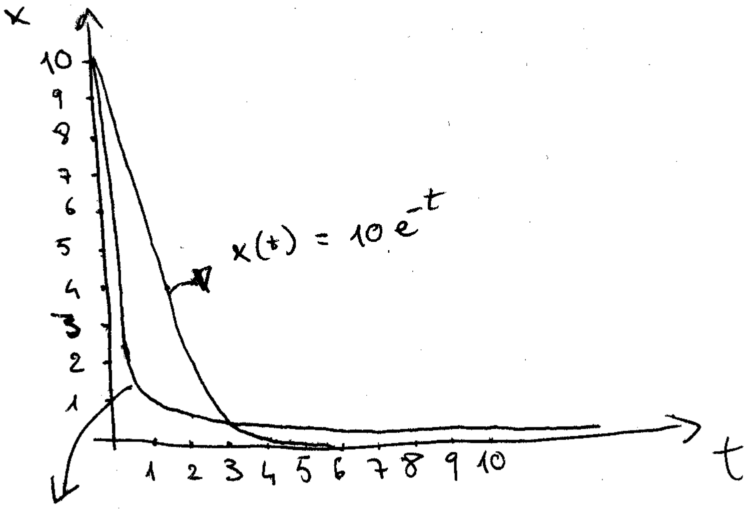
$$\Rightarrow x^2 = \frac{1}{2(t+C_1)}$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{2(t+C_1)}}, \quad C_1 = \frac{1}{2x_0^2}$$

b) On the other hand

$\dot{x} = -x \Rightarrow x = x_0 e^{-t}$

2.4.9



$$x = \frac{1}{\sqrt{2t + 1/100}}$$