9.3.1 HW # 13

a) All the trajectories never escape the torus $[0, 2\pi] \times [0, 2\pi]$, so that ~~the~~ they cannot separate exponentially fast.

b) A system w/ no chaos has zero Liapounov exponents in either time direction. Let's check it for our system

$$\begin{cases} \dot{\theta}_1 = \omega_1 \\ \dot{\theta}_2 = \omega_2 \end{cases} \Rightarrow \begin{cases} \theta_1 = \omega_1 t + C_1 \\ \theta_2 = \omega_2 t + C_2 \end{cases}$$

$$\vec{\eta}(t) = \left|\vec{\theta}^{(1)}(t) - \vec{\theta}^{(2)}(t)\right| = \left|(\omega_1 t + C_1, \omega_2 t + C_2) - (\omega_1 t + C_3, \omega_2 t + C_4)\right| = \left|(C_1 - C_3, C_2 - C_4)\right|$$

$$= \sqrt{(C_1 - C_3)^2 + (C_2 - C_4)^2}$$ which is also equal to

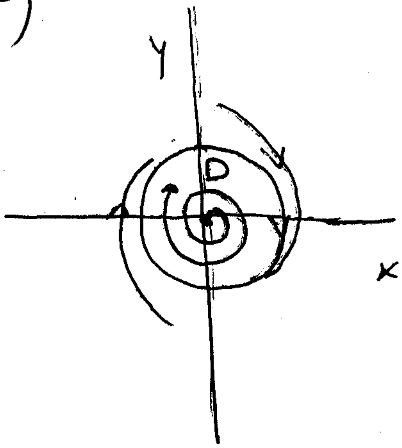
$$\vec{\eta}(0) = \left|(C_1, C_2) - (C_3, C_4)\right|$$

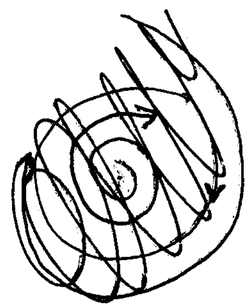
therefore $\ln \left|\dfrac{\vec{\eta}(t)}{\vec{\eta}(0)}\right| = \ln 1 = 0$

hence $\displaystyle\lim_{t \to \infty} \frac{1}{t} \ln \left|\frac{\vec{\eta}(t)}{\vec{\eta}(0)}\right| = 0$

9.3.8

$$\begin{cases}\dot{r} = r(1-r^2)\\ \dot{\theta} = 1\end{cases}$$



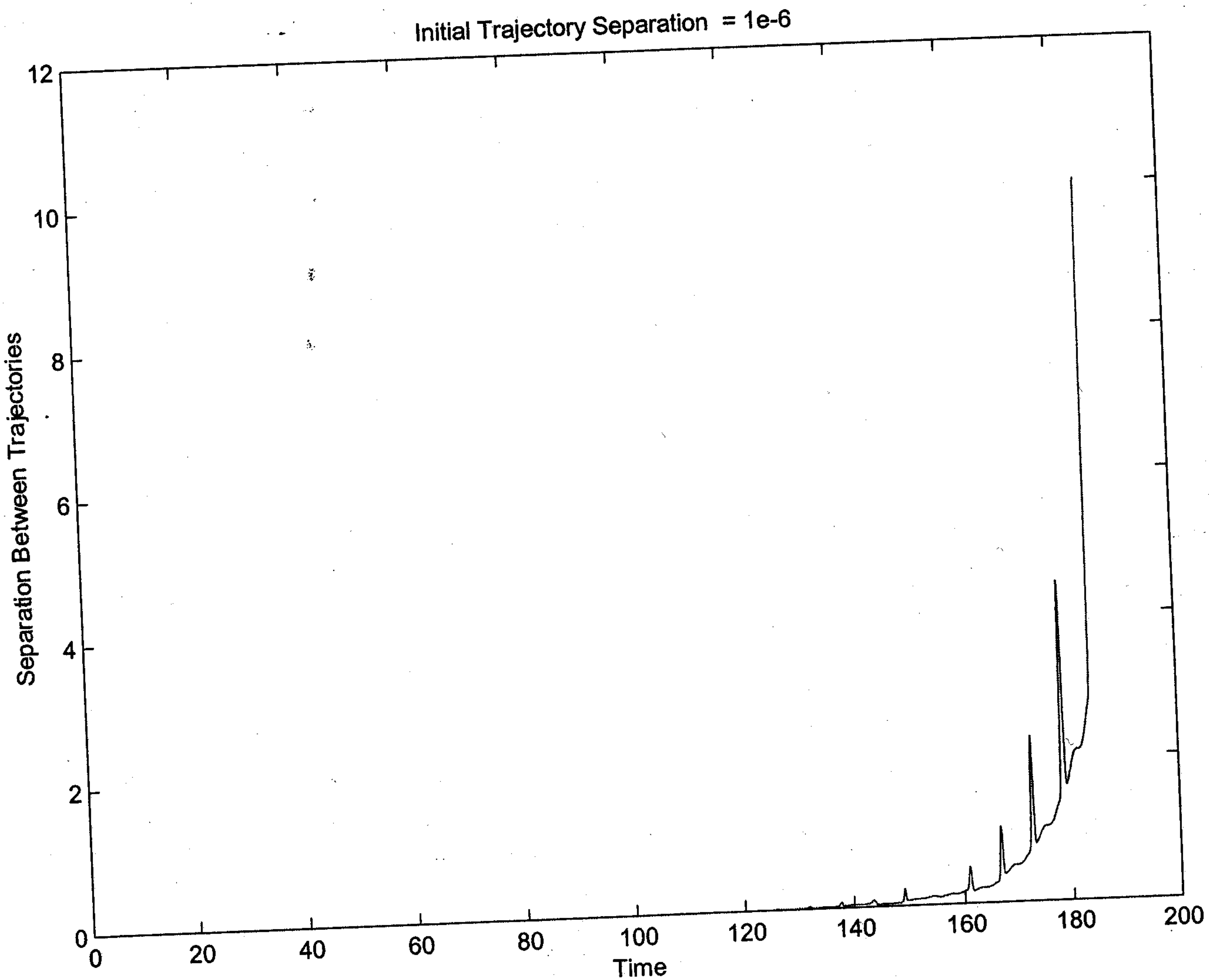


a) A trajectory that starts in D either ends up in the stable cycle or if it starts at the origin it stays there. Since $D = \{(x,y)\in \mathbb{R}^2 : x^2+y^2 \leq 1\}$ D is an invariant set.

b) All trajectories in the x-y plane either ends in the limit cycle or in the fixed point, therefore D attracts an open set of initial conditions ($\mathbb{R}^2$ is the open set)

c-d) D is not minimal, though: its subset $\{(x,y)\in \mathbb{R}^2 : x^2+y^2=1\}$ (the limit cycle) is invariant an attracts an open set of initial conditions, that is $\mathbb{R}^2 \setminus \{(0,0)\}$, which is its basin of attraction. Hence the limit cycle is an attractor for our system.

Problem 3: the ~~size~~ size of the attractor has a lengthscale of about 10, therefore that size is reached after about 180 time units.

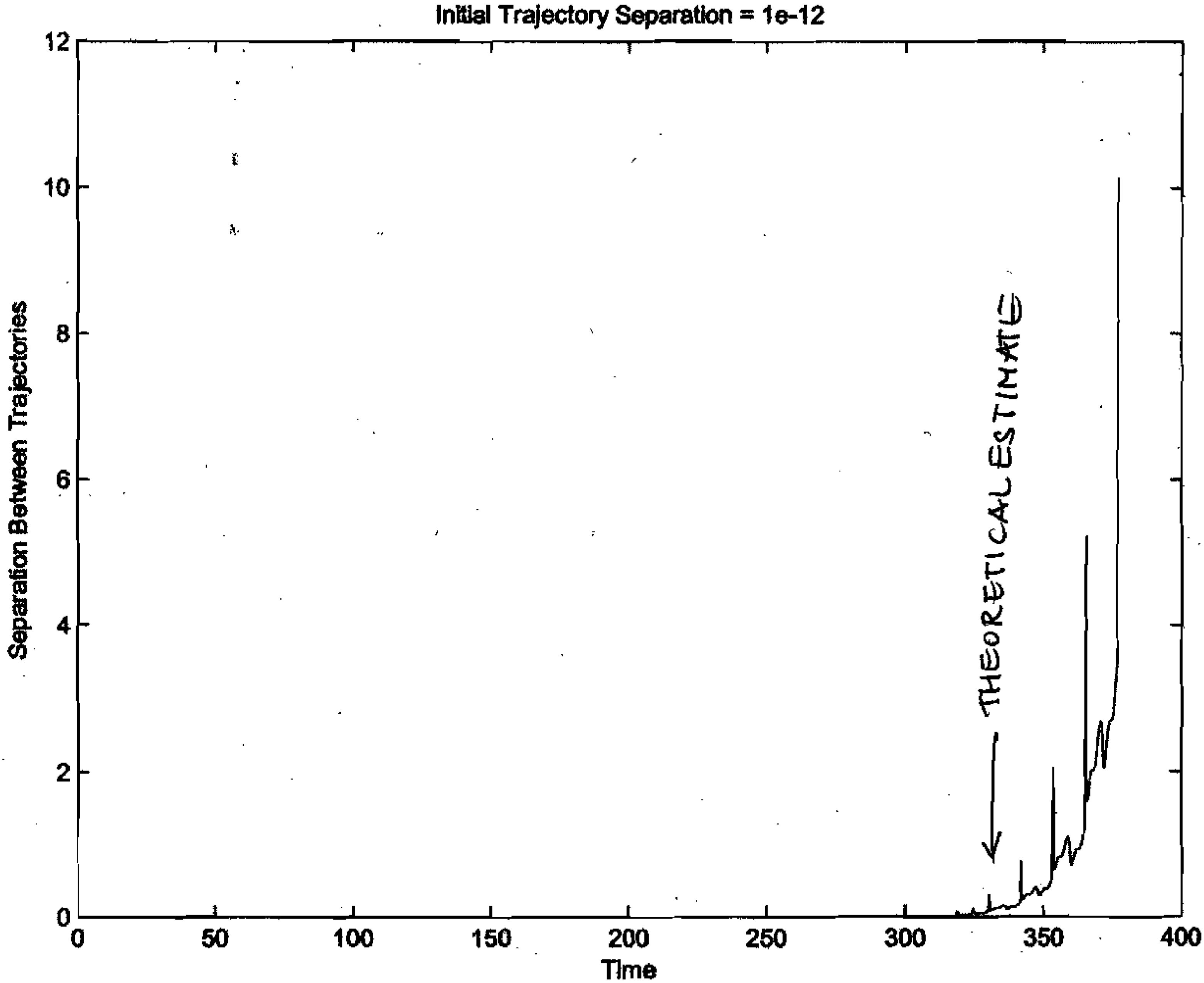


b) From $\delta(t) = \delta(0) e^{\lambda t}$ we find $\lambda = \frac{1}{t} \ln \frac{\delta(t)}{\delta(0)}$,

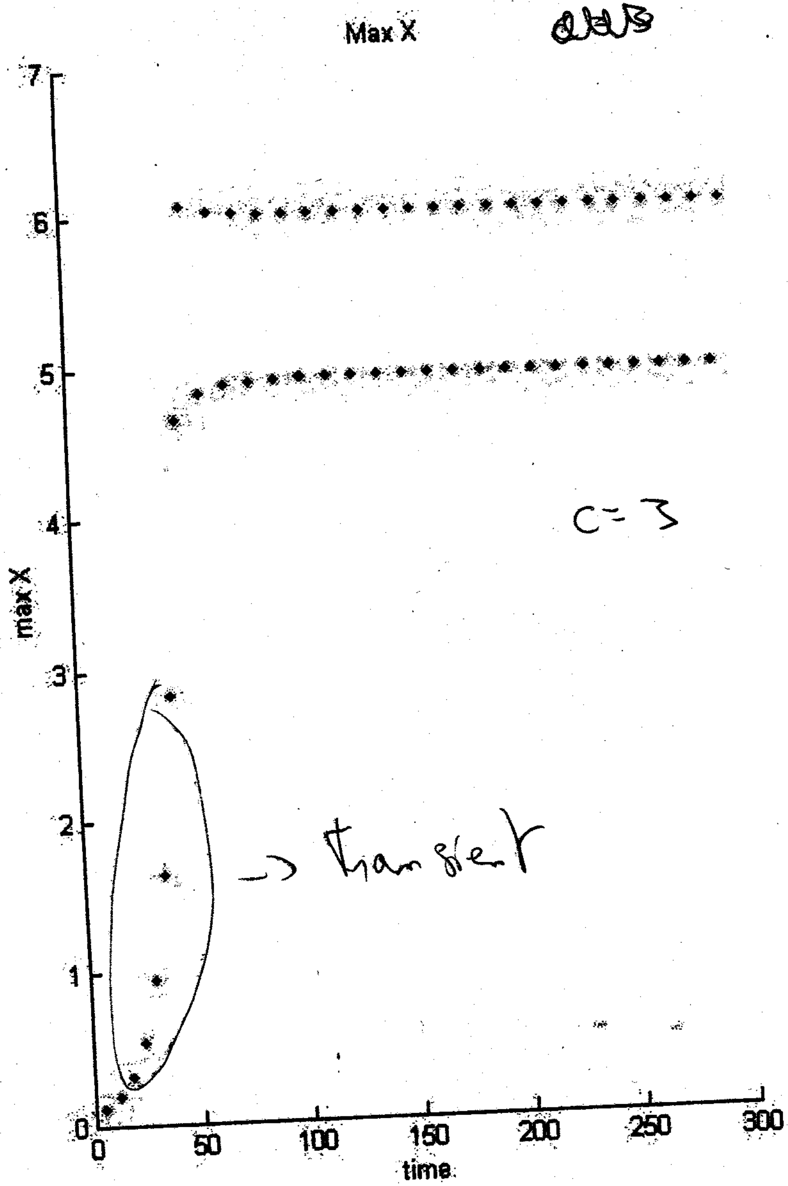
so from part a) $\lambda \approx \frac{1}{180} \ln 10^7 \approx 0.09$

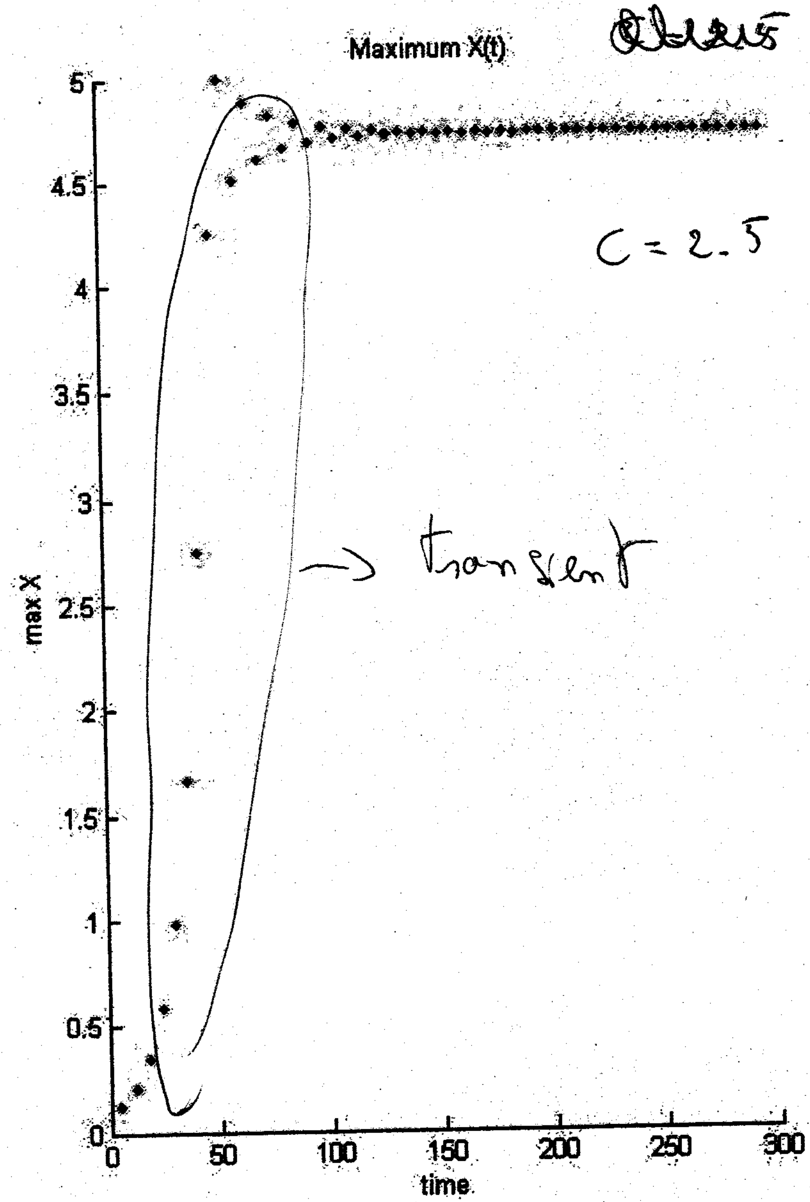
For $\delta(0) = 10^{-12}$ we should expect

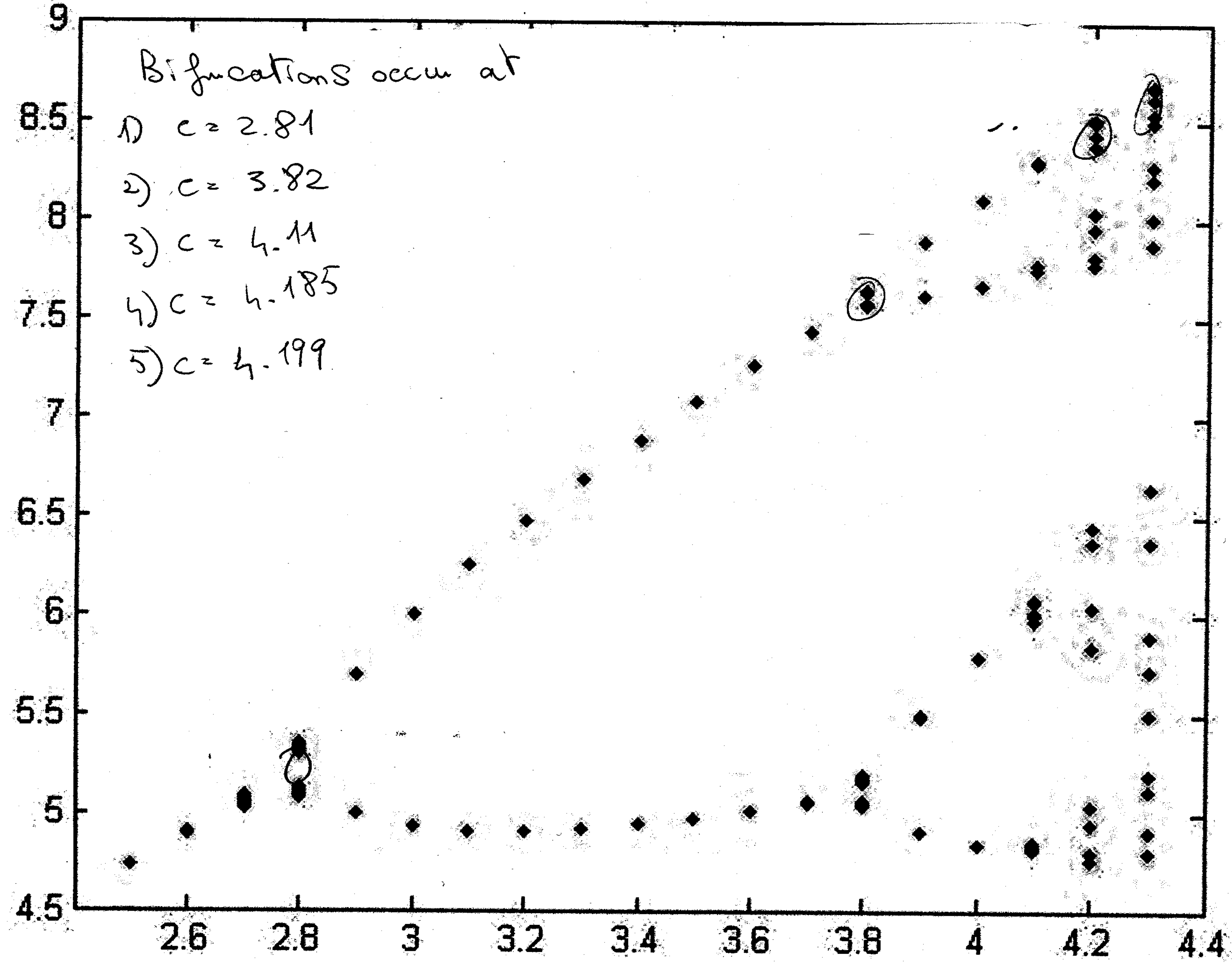
$t \approx \frac{1}{\lambda} \ln \frac{\delta(t)}{\delta(0)} \approx \frac{1}{0.09} \ln 10^{13} \approx 335$



problem 4.


Max X
C = 3
→ transient
max X
time
0
1
2
3
4
5
6
7
50
100
150
200
250
300


Maximum X(t)
C = 2.5
→ transient
max X
time
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
50
100
150
200
250
300


Bifurcations occur at
1) c = 2.81
2) c = 3.82
3) c = 4.11
4) c = 4.185
5) c = 4.199
9
8.5
8
7.5
7
6.5
6
5.5
5
4.5
2.6
2.8
3
3.2
3.4
3.6
3.8
4
4.2
4.4