Problem 1

Rayleigh-Bénard convection with poorly conducting boundaries. Consider the equation (which actually describes free fluid convection when the top and bottom plates are poor thermal conductors)

$$\partial_t u = ru - (1+\nabla^2)^2 u + \nabla \cdot [(\nabla u)^2 \nabla u]$$

for a real field $u(x, y, t)$ in a two dimensional space.

(a) What is the growth rate $\sigma_{\mathbf{q}}$ of a small perturbation at wave vector $\mathbf{q}$ from the uniform state $u = 0$? What is the critical wave number q_c and critical value of the control parameter r_c giving the primary instability as r is raised?

(b) For a nonlinear saturated *stripe* solution

$$u = a \cos x + \cdots$$

find out how the amplitude a varies with $r - r_c$ near threshold to lowest non-trivial order.

(c) Repeat the calculation for a *square* solution

$$u = a(\cos x + \cos y) + \cdots .$$

In these expressions the $\cdots$ denote higher order terms.

Problem 2

Stability of stripe and square patterns. Show using a Galerkin method truncated to the lowest order modes, that the square state in Problem 1 is stable (you only need to consider perturbations which correspond to the growth or decay of one set of stripes relative to the other). Next show that the stripe state is unstable towards the growth of an orthogonal set of stripes (cross-roll instability), i.e., the stripe state is unstable towards squares.

Problem 3

Eckhaus instability. Calculate the stability of the stripe solutions to the Swift-Hohenberg equation near onset to a longitudingal perturbation in the one mode approximation, following the method we used in class to consider the stability with respect to transverse perturbations (zigzag instability).