We have found that the amplitude equation for a two-dimensional uniaxial system is

\[ \tau \partial_t A = \epsilon A + \xi_x \partial_x^2 A + \xi_y \partial_y^2 A - g |A|^2 A, \]

where \( \tau, \epsilon, \xi_x, \xi_y, \) and \( g \) are positive constants.

(a) Nondimensionalize the equations to reduce the number of parameters to a minimum and therefore determine the scaling of \( A, x, y, \) and \( t. \)

(b) Do the linear stability analysis of this equation about the uniform state \( A = 0. \) Since \( A \) is complex, you will have to expand it, writing two coupled equations, one for the real and one for the imaginary part of \( A. \) (Hint: group the real terms separately from the imaginary terms in the amplitude equation after substitution \( A = U + iV). \)

(c) The amplitude \( A \) depends on just two spatial coordinates, \( x \) and \( y. \) Argue that the field \( u \) of which \( A(x, y, t) \) is the amplitude also has to depend on a spatial coordinate other than \( x \) and \( y \) (we can call the extra coordinate \( z \)), i.e., \( u = u(x, y, z, t). \)

(d) What is the instability type for \( A \) that you found in part (b)? What is the type of the corresponding instability for the field \( u? \) Explain why you get the same (or different) answer.

(e) Suppose this amplitude equation is solved on a rectangular domain \(-L_x < x < L_x, -L_y < y < L_y\) with suppressing boundaries. What are the corresponding boundary conditions on \( A \) on each of the boundaries?

(f) Given these boundary conditions, what is the distance over which the amplitude adjusts from the value at the boundary to the value in the bulk in the scaled and unscaled variables?

(g) What is the nonlinear saturated amplitude in the bulk (far enough away from the boundaries) in the scaled and unscaled variables?

(h) Do the linear stability analysis of the amplitude equation about the ideal pattern state \( A \propto \exp(iK \cdot x) \) assuming a laterally infinite system. What does this result tell about the stability balloon of the ideal pattern \( u(x, t)? \) What is the region of validity of this linear stability analysis?